Instructions: Try these problems from the textbook as practice (answers below).

Problems

2-10. Compute $\mathbf{a} \cdot \mathbf{b}$.

3. $\mathbf{a} = \langle 1.5, 0.4 \rangle$, $\mathbf{b} = \langle -4, 6 \rangle$
5. $\mathbf{a} = \langle 4, 1, 1/4 \rangle$, $\mathbf{b} = \langle 6, -3, -8 \rangle$
7. $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j}, \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$
9. $|\mathbf{a}| = 7, |\mathbf{b}| = 4$, the angle between $\mathbf{a}$ and $\mathbf{b}$ is $30°$.

13. (a) Show that $\mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = 0$.
    (b) Show that $\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1$.

15-20. Compute the angle between the given vectors exactly.

15. $\mathbf{a} = \langle 4, 3 \rangle$, $\mathbf{b} = \langle 2, -1 \rangle$
17. $\mathbf{a} = \langle 1, -4, 1 \rangle$, $\mathbf{b} = \langle 0, 2, -2 \rangle$
19. $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}, \mathbf{b} = 2\mathbf{i} - \mathbf{k}$

21. Find the three angles of the triangle with the vertices $P(2, 0)$, $Q(0, 3)$, and $R(3, 4)$.

23. Determine whether the given vectors are orthogonal, parallel, or neither.

    (a) $\mathbf{a} = \langle 9, 3 \rangle$, $\mathbf{b} = \langle -2, 6 \rangle$
    (b) $\mathbf{u} = \langle 4, 5, -2 \rangle, \mathbf{v} = \langle 3, -1, 5 \rangle$
    (c) $\mathbf{a} = -8\mathbf{i} + 12\mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = 6\mathbf{i} - 9\mathbf{j} - 3\mathbf{k}$
    (d) $\mathbf{a} = 3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = 5\mathbf{i} + 9\mathbf{j} - 2\mathbf{k}$

25. Use vectors to decide whether the triangle with vertices $P(1, -3, -2)$, $Q(2, 0, -4)$ and $R(6, -2, -5)$ is right-angled.

27. Find a unit vector that is orthogonal to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$.

29. Find the acute angle between the lines $2x - y = 3$ and $3x + y = 7$.

31. Find the acute angles between the curves $y = x^2$ and $y = x^3$ at their points of intersection. The angle between two curves is the angle between their tangent lines at the point of intersection.

33-37. Find the direction cosines and direction angles of the vector.

33. $\langle 2, 1, 2 \rangle$
35. $\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$
37. $\langle c, c, c \rangle$ where $c > 0$

39-44. Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$.

39. $\mathbf{a} = \langle -5, 12 \rangle, \mathbf{b} = \langle 4, 6 \rangle$
41. $\mathbf{a} = \langle 4, 7, -4 \rangle, \mathbf{b} = \langle 3, -1, 1 \rangle$
43. $\mathbf{a} = 3\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$

47. If $\mathbf{a} = \langle 3, 0, -1 \rangle$, find a vector $\mathbf{b}$ such that $\text{comp}_{\mathbf{a}}(\mathbf{b}) = 2$.

Answers

3. -3.6
5. 19
7. 1
9. $14\sqrt{3}$
15. $\arccos(1/\sqrt{5})$
17. $\arccos(-5/6)$
19. $\arccos(7/\sqrt{130})$
23. orthogonal; neither; parallel; orthogonal
25. yes
27. $\pm(\mathbf{i}-\mathbf{j}-\mathbf{k})/\sqrt{3}$
29. 45°
33. $2/3$, $1/3$, $2/3$
35. $1/\sqrt{14}$, $-2/\sqrt{14}$, $-3/\sqrt{14}$
37. $1/\sqrt{3}$, $1/\sqrt{3}$, $1/\sqrt{3}$
39. 4, $\langle -20/13, 48/13\rangle$
41. $1/9$, $\langle 4/81, 7/81, -4/81\rangle$
43. $-7/\sqrt{19}$, $-\dfrac{21}{19}\mathbf{i}+\dfrac{21}{19}\mathbf{j}-\dfrac{7}{19}\mathbf{k}$
47. any vector of the form $\left\langle s, t, 3s-2\sqrt{10}\right\rangle$, $s,t\in\mathbb{R}$